Build a Sierpinski Pyramid

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Students from Anoka High School, located in Anoka, Minnesota, built a nineteen-foot-tall Sierpinski pyramid in the Minneapolis Convention Center in conjunction with the NCTM's 75th Annual Meeting in April 1997. This activity was part of a unit on fractal geometry.

BEGINNING THE ACTIVITY

The fractal unit focused primarily on fractals that could be created by hand. The unit began by detailing the basic characteristics of most fractals, self-similarity and iteration. Self-similarity can be described as "a part looks like the whole, a part of the part looks like the part," and so on. Iteration is the repetition of a process, with the output of one stage becoming the input for the next. These characteristics are shown using some famous fractals, including those that follow, which are drawn at stages 0, 1, and 2.

The fractal in **figure 1** is called *Cantor dust,* after the mathematician Georg Cantor. The iteration in this fractal is the trisection of a segment and the removal of the middle one-third at each stage. The name is appropriate because little is left but "dust" after a few more stages.

The process used for *Purina Dog Chow* (see **fig. 2**) begins with a completely shaded square. Trisect each side of the square, and connect the points of trisection parallel to the sides of the square, as if making a tick-tack-toe grid. Keep the four "corner" squares and the "middle" square, and but remove the others. Continue this process at each stage.

Fig. 1
Cantor dust (stages 0–2)

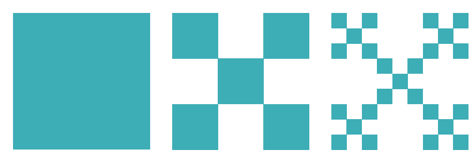

Fig. 2
"Purina Dog Chow" (stages 0–2)

The fractal shown in **figure 3** is the *Koch snowflake,* named after Helge von Koch. Start with an equilateral triangle, and trisect each side. Remove the middle one-third of each side, and replace it with two segments congruent to the missing piece.

Fig. 3
Koch snowflake (stages 0–2)

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Figure 4 shows the *Sierpinski triangle,* or *Sierpinski gasket,* named for Waclaw Sierpinski. Start with a completely shaded equilateral triangle, and connect the midpoints of the sides forming four smaller, congruent equilateral triangles. Remove the middle equilateral triangle. Continue this iteration on each of the remaining shaded equilateral triangles. The Sierpinski pyramid is based on this fractal.

Fig. 4
Sierpinski gasket (stages 0–2)

THE SIERPINSKI PYRAMID

The Sierpinski pyramid constructed in the Minneapolis Convention Center was the culmination of work done over a period of five years by Anoka High School students. The first pyramid, built in April 1993, was more than nine feet tall. That pyramid remained upright overnight but toppled the next day. The following year, a pyramid of the same size lasted three days before toppling. In 1995, new construction techniques enabled the students to build a pyramid that stood until they decided to take it down. After this successful construction, we decided to save the pieces of the pyramid and try to construct one that would be twice as tall. To accomplish this feat, students would have to build and save four pyramids and put them back together in a place that had a ceiling at least nineteen feet high. Because we knew that NCTM's 1997 annual meeting would be held in Minneapolis, just twenty-five miles from Anoka High School, our goal was set to put up the nineteen-foot pyramid at that meeting. We received permission from NCTM, John Erickson and the local arrangements committee, the Minneapolis Convention Center, and the fire marshal. We scheduled construction for Monday, 14 April, three days before the 75th Annual Meeting began.

Because we had not completely broken down the pyramids but had saved them as stage 2 pyramids, the process at the Convention Center was expedited. Thirty students arrived at the Convention Center at 10:00 A.M. on Monday, 14 April, and finished building the pyramid at 6:30 P.M. on the same day. The final stage was put together with a cherry picker on loan from NCTM, which had rented it for use in the Exhibit Hall.

CONSTRUCTING A SIERPINSKI PYRAMID

1. Make enough copies of the pyramid template to construct a pyramid of the desired size. **Table 1** shows the number of templates needed for each stage. A thick, card-stock type of paper should be used; we used sixty-seven-pound paper in Minneapolis. **Figure 5** shows a sample of the template. Note that the approximate height of the pyramid depends on the size of the individual template.

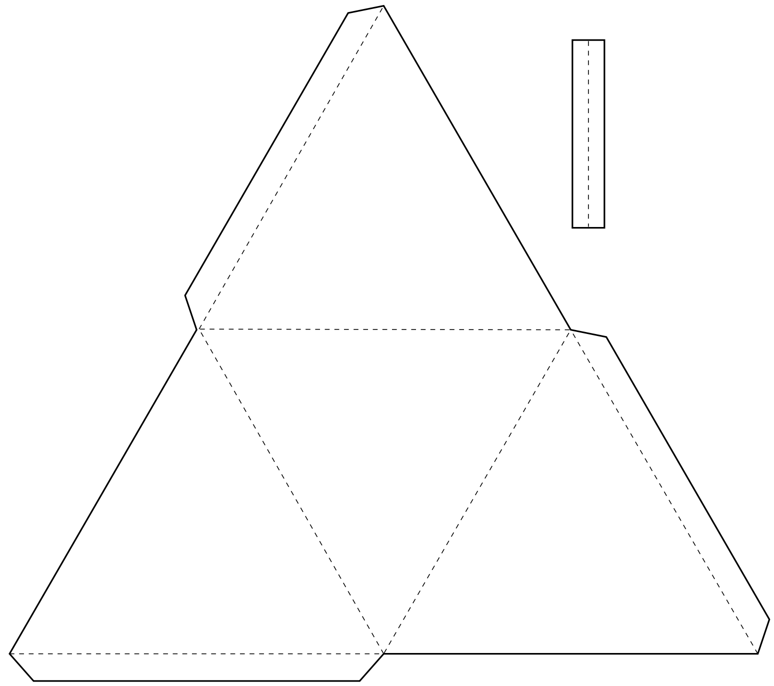

Fig. 5
A template and brace for making a pyramid

TABLE 1
Number of Templates Needed for Sierpinski Pyramid

Stage	Number of Templates	Approximate Height (Inches)
0	1	3.5
1	4	7
2	16	14
3	64	28
4	256	56
5	1024	112
6	4096	224

2. Cut out each template. Be sure to save the leftover scraps, as they will be needed later.

3. Fold each template to form a small pyramid. Tape the edges together, but leave approximately one inch of space at each vertex. For lack of a better term, we call this construction a stage 0 pyramid.

4. Put four of these stage 0 pyramids together to form a pyramid with the "middle" missing. Make a small rectangular brace from the material left over after cutting out the templates, and glue it on an edge to hold the vertices together (see **fig. 6**). This structure is a stage 1 pyramid.

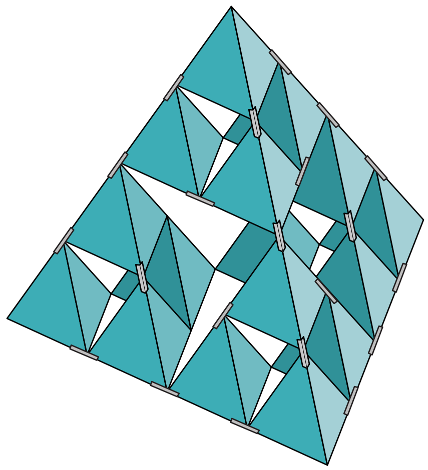

Fig. 6
The braces give rigidity from one level to another.

5. Put four stage 1 pyramids together in the same manner as you assembled the stage 1 pyramid, to form a stage 2 pyramid.

6. At this point, you will have to move your materials to the site where the completed Sierpinski pyramid will eventually reside, because after one more stage the pyramid will probably be too big to fit through most doorways.

7. Continue the process until your Sierpinski pyramid reaches the desired size. A stage 5 pyramid will have a base that is an equilateral triangle approximately 140 inches on a side.

Materials needed

- Pyramid templates—the number needed depends on the size of the pyramid that you plan to build.
- Tape—approximately twenty rolls of transparent tape for a stage 5 pyramid
- Scissors—one pair for each student
- Glue—a bottle of glue for each group
- If you make a stage 6 pyramid, wood or metal reinforcement might be necessary to glue to the edges for additional support (we used corner bead in Minneapolis)

Helpful hints or ideas

1. Before starting the project, explain to the students exactly what you are doing, as well as the step-by-step procedure, to help them see the big picture.

2. More than one class should work on the project. It takes approximately ten to twelve 47-minute class periods to complete a stage 5 pyramid. If, for example, three classes work on the project, one class can continue where the previous class left off, thus reducing the amount of time that each class spends on the project.

3. When the pyramid is complete, remind the students that the actual Sierpinski pyramid is formed by *removing* pieces of an original pyramid rather than by piecing together individual, smaller pyramids. However, since the "real" process is impractical for a classroom construction, the method in this article is most convenient.

4. Print more templates than you need, because some will be cut improperly, others will get crushed, and so on.

5. To save time, divide the number of templates by the number of students working on the project, and have students cut out the templates the night before construction begins. Be sure that the students bring back the cut templates along with the extra pieces, since you will need the extra card stock for the braces.

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The author would like to thank the late Lee Yunker and the NSF-sponsored geometry projects at Saint Olaf College for their inspiration in constructing the pyramid.