Tetrahedral Kites

Each student constructs a tetrahedron and describes the linear, area and volume using non-traditional units of measure. Four tetrahedra are combined to form a similar tetrahedron whose linear dimensions are twice the original tetrahedron. The area and volume relationships between the first and second tetrahedra are explored, and generalizations for the relationships are developed.

Learning Objectives

Students will:

- Construct a tetrahedral puzzle
- Construct a tetrahedron from straws, thread, and tissue paper
- Combine four tetrahedra to make a larger tetrahedron
- Calculate the surface area and volume of 3-D shapes
- Discover the relationship between the linear, area, and volume measures of similar polyhedra

Materials

For each group of four students:

- 24 eight-inch straws
- Kite string or crochet thread
- Glue stick
- Tissue paper, 20" × 26"
- Scissors
- Tape
- Tissue Paper Template (4 copies)
- Length, Area, and Volume Activity Sheet (for Assessment)

Instructional Plan

In this lesson, students work together in groups to create tetrahedron models using straws, string, and tissue paper. The models are then combined to make larger tetrahedrons, and students compare the ratios of edge length, area, and volume of the models in an attempt to understand scaling factors.

To begin the lesson, organize your class into cooperative groups of four students. To create a bond among the students in each group, begin with the tetrahedron puzzle. Give each group four copies of the Tissue Paper Template, two of one color, and two of a different color. Instruct the students to cut around the perimeter of the template. Fold the piece on each of the segments. Match each of the tabs with their corresponding letter. Use scotch tape to tape the tabs on the outside of the corresponding sides. Instruct each group to use two pieces, one of each color, to put the two three-dimensional pieces together in some way so that the completed three-dimensional object will have four congruent equilateral triangular faces. The finished object is a tetrahedron. Allow students time to explore possible ways to put the two pieces together. Many will eventually discover that the square faces can't be part of an equilateral triangle and can be hidden by putting the two square faces together. If this doesn't form the tetrahedron, then ask if the two square faces can be put together in another way. By rotating one of the pieces 90°, the tetrahedron will be formed. Set the model aside as it will be referred to later.

Tissue Paper Template

Each group leader should then distribute six straws. Instruct the students to use three straws to make an equilateral triangle, so that the length of each side of the triangle is one straw. Students will have no trouble with this. Then, tell them to use two more straws to make two congruent equilateral triangles with the five straws so that the length of each side of the triangles is one straw. Again, students will have no trouble with this. Ask them to describe the shape they have made. Responses should include two equilateral triangles, a parallelogram, and a rhombus. Use the last straw with the other five to make four congruent equilateral triangles so that the side length each triangle is one straw. This is more difficult because the students try to do it in a plane. Eventually, however, a student will realize that the object can be three-dimensional and will construct a tetrahedron. To help the process along, refer the students to the tetrahedral puzzle they have in front of them.

After students complete this opening puzzle, they will move to the main portion of the lesson, in which they build tetrahedrons. Constructing one tetrahedron involves the following steps:

1. String three straws on the 40" string. Tie the string so that the three straws form a tight triangle. It is important that one end of the remaining string be long enough to pass through another straw. The easiest way to pass the string through the straw is to start the string into the straw and then suck the string through the straw.
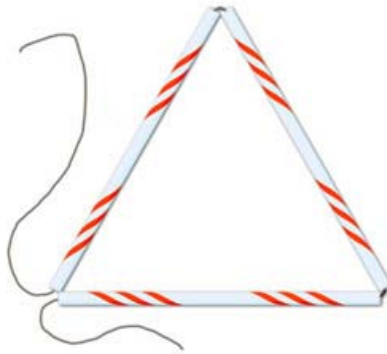
2. Tie en end of each of the two short strings to the vertices where there is no knot. Then, pass each string through a straw and tie the ends together to form a tight triangle. The five straws should now form a rhombus with one diagonal, as shown below.
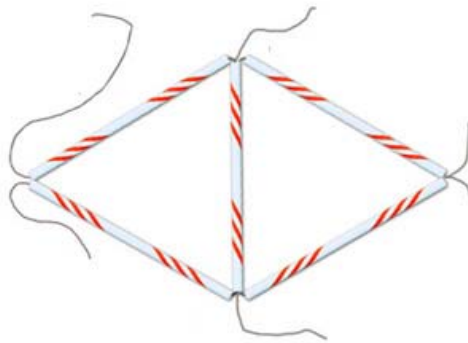
3. Pass the long string through the last straw and connect it to the opposing vertex to form a tetrahedron. Tie the ends together to complete the shape.
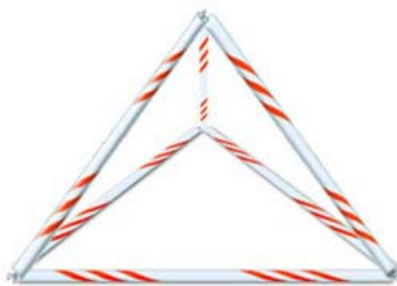
4. Fold a 20" × 26" piece of tissue paper into fourths, as shown. Each quarter will be 10" × 13" and is enough to cover two faces of the tetrahedron. Fold the 10" × 13" pieces into fourths, and place the [template](#) on two folded edges. Cut along the three sides marked CUT. (Students should place the template and make the three cuts while the tissue paper is still folded. This will prevent unnecessary cutting, and the proper shape will result when the paper is unfolded.)
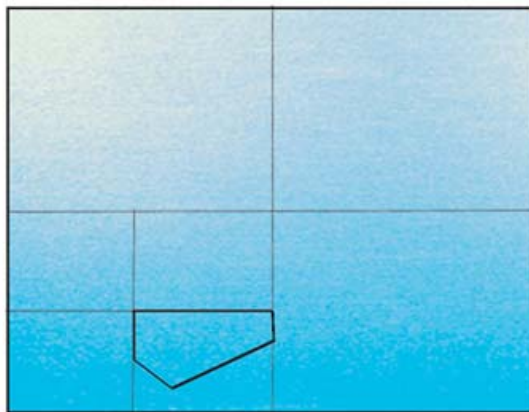
5. Unfold each quarter. They will look like this:
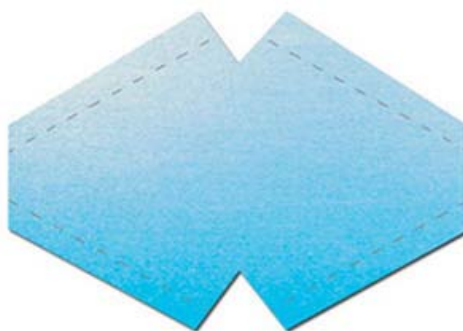
6. Attach the tissue paper to two faces of the tetrahedron. Lay the tissue paper on the table. Put glue along the flaps, and fold the flaps over the straws. Rotate the tetrahedron, apply glue to the other flaps, and fold these flaps over the straws.
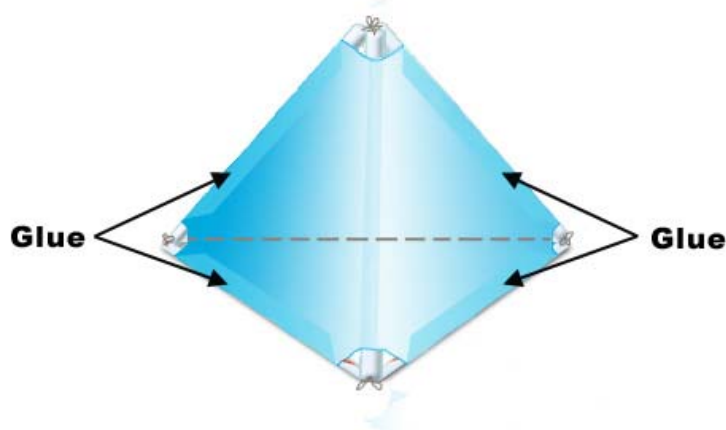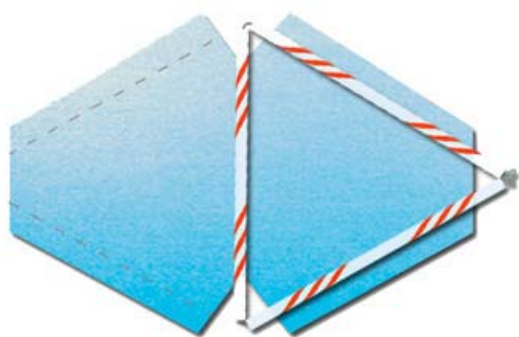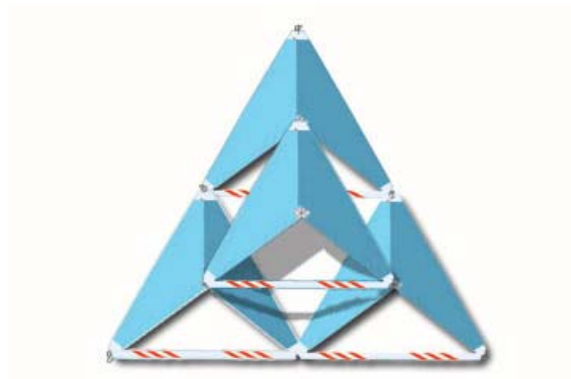

Call the resulting shape Tetrahedron 1. Tell students that the edge length is one straw, the area of a face is one triangle, and the volume is one tetrahedron.

7. Have students combine four copies of Tetrahedron 1 to make a larger tetrahedron. Place three tetrahedra on the table as shown, and tie the adjacent base vertices together.
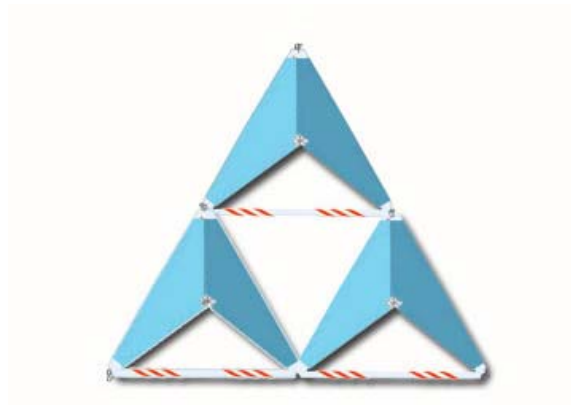
8. Place the fourth tetrahedron on top of the other three, and tie the adjacent vertices. Make the joints tight with no slack in the string for a sturdier model.

Call the resulting larger shape Tetrahedron 2. The final result will look like the figure shown below.
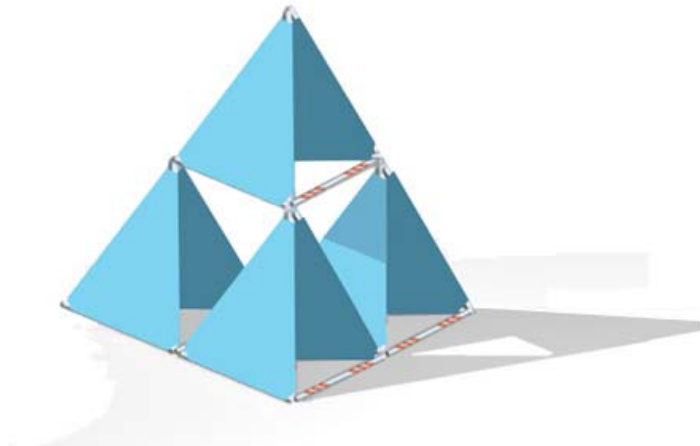
Using 16 copies of Tetrahedron 1, or four copies of Tetrahedron 2, students can create a larger version of the tetrahedron model. Shown below is the model created by students from the 2005 Summer Honors Program in Holdrege, NE.

Once the models are complete, students will compare the edge length, area, and volume of Tetrahedron 1 and Tetrahedron 2. Ask the questions in the Questions For Students section below. This discussion is the primary focus of the lesson, and students should be given ample time to discuss each question with the members of their group.

To help students determine the volume, have each group make four tetrahedron pieces and one octahedron piece. Use these pieces to make a tetrahedron with the octahedron in the center. This helps to verify that the open space in the center of Tetrahedron 2 is an octahedron. What is the volume of the octahedron in terms of Tetrahedron 1? Cut the octahedron into two square pyramids; then, cut one of the square pyramids in half by making the cut from the diagonal of the square base to the opposite vertex creating two triangular pyramids each made up of two isosceles right triangles and two equilateral triangles. Compare the equilateral bases and the heights of the tetrahedron and the triangular pyramid. What can you say? The triangular bases and heights of the tetrahedron and the triangular pyramid are equal. What can you say about the volumes of these two pieces? The volumes are also equal. What can you say about the volume of the octahedron? The volume of the octahedron is equivalent to four times the volume of Tetrahedron 1, so the volume of Tetrahedron 2 is equivalent to eight times the volume of Tetrahedron 1.

Questions for Students

Compare a face of Tetrahedron 1 with a face of Tetrahedron 2. Describe what you see.

> [The face of Tetrahedon 1 is one equilateral triangle. The face of Tetrahedron 2 is made up of four equilateral triangles, each congruent to one face of Tetrahedron 1.]

How do you know that the triangular face of Tetrahedron 1 similar to the triangular face of Tetrahedron 2?

> [Each of the corresponding edge lengths is twice as long, and the corresponding angles are congruent.]

What are the ratios of edge length and area from Tetrahedron 1 to Tetrahedron 2?

> [Edge length, 1:2. Area, 1:4.]

Start with a unit square with an edge length of 1. Arrange four unit squares together to make a larger square. What is the ratio of the edge lengths of these squares. What is the area ratio of these squares?

> [Edge length, 1/2. Area, 1/4.]

Make a conjecture about the relationship between the edge length ratio and the area ratio of similar figures.

> [The area ratio of similar figures is the square of the edge length ratio. Test this conjecture with other similar polygons.]

What is the volume of Tetrahedron 2 in terms of Tetrahedron 1?

> [The volume of Tetrahedron 2 is eight times the volume of Tetrahedron 1. Allow your students time to discuss their answer to this question. It is not trivial. Most students will say 6, and some will say 7, but rarely will any say 8.]

Begin with a unit cube. Build a similar cube by doubling the length of each edge. What is the volume of the new cube in relation to the unit cube?

> [The volume of the new cube is eight times the volume of the original cube.]

To determine the volume of Tetrahedron 2, first identify the shape of the polyhedron that fills the open space bounded by the planes of the faces of Tetrahedron 2 and the faces of Tetrahedron 1. What is the shape of the polyhedron that will fill the open space?

> [The open space is an octahedron. Your students should find the square in Tetrahedron 2 that corresponds to the squares where the

two puzzle pieces were put together. Place a piece of paper on the square that is in the open space. This square is the base of two square pyramids. Each face of thes pyramids is an equilateral triangle. When the two square pyramids are joined at the square base, an octahedron is formed.]

Assessment Options

1. In whole-class discussions, ask students to describe the relationship between linear, area, and volume ratios of similar figures and give examples to validate their claims. Encourage and validate a variety of appropriate responses.
2. Ask students to write an entry in their journals that includes the following pieces:

    - A summary of what they found regarding the relationship between linear, area, and volume ratios of similar figures.
    - Examples that support their summary.
    - Any tables, charts, or other tools they used to organize their information.
3. Have students complete the problems on the Length, Area, Volume sheet.

Length, Area, Volume Activity Sheet

You can review the solutions to this worksheet with students.

Extensions

1. Explore the relationships between the formulas for finding the volumes of prisms and pyramids.
2. Combine four copies of Tetrahedron 2 to make Tetrahedron 4, and then combine four copies of Tetrahedron 4 to make Tetrahedron 16. Use these models to explore the relationship between edge length ratios, area ratios, and volume ratios of similar figures with various scale factors. Can students explain the names given to each tetrahedron model?

Teacher Reflection

- How did your lesson address auditory, tactile and visual learning styles?
- Were concepts presented too abstractly? Too concretely? How would you change them?
- What worked with classroom behavior management? What didn't work? How would you change what didn't work?
- Did you challenge the achievers? How?
- What content areas did you integrate within the lesson? Was this integration appropriate and successful?
- How did your lesson address auditory, tactile and visual learning styles?

NCTM Standards and Expectations

Geometry 6-8

1. Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
2. Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.

Geometry 9-12

1. Analyze properties and determine attributes of two- and three-dimensional objects.
2. Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them
3. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.
4. Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

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